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HEATING OF A FLANGED JOINT

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The problem of heating of a flanged joint is treated in a general manner, allowing for the thermal stresses. An approximate method of solution is proposed.

If a flange is considered as a plate with one dimension substantially less than the other two, the thermal stresses at points sufficiently far from the edges may be determined from the formula for an infinite plate [1]

$$\sigma = -\frac{\beta E}{1-\nu}\, t + \frac{\beta E}{2R(1-\nu)} \int_{-R}^{R} t dx + \frac{3\beta E x}{2R^3(1-\nu)} \int_{-R}^{R} t x dx.$$

It is not hard to see that given a linear variation with time of the temperature at all points of the plate (boundary condition of the second kind) and constant thermal conductivity, the temperature distribution through the plate after a certain time will be described by a quadratic parabola of constant shape, and the thermal stresses will not vary with time.

The temperature field in the plate may also be described approximately for other boundary conditions [2].

The duration of the first phase, when the temperatures at some section through the thickness of the plate are practically indistinguishable from their initial values, may be shown by calculation to be comparatively short and measurable in minutes. This phase may therefore be excluded from consideration, especially since the temperature differences and the stresses do not attain their maximum values in this period. Then

$$t = f(\tau) + t_0 + k(x - x_{\min})^2/2. \tag{1}$$

If we take β, E, ν to be independent of temperature, the stresses in the plate for a quadratic distribution of temperature through the thickness will be

$$\sigma = \frac{\beta E}{1-\nu}\left[-\frac{kx^2}{2} + \frac{kR^2}{6}\right]$$

for any value of $x_{\min}$.

The heat flux per unit surface area of the flange in unit time is

$$dq = \left.\frac{\partial t}{\partial x}\right|_{x=R} \lambda - \lambda \left.\frac{\partial t}{\partial x}\right|_{x=-R} = 2kR$$

and is also independent of $x_{\min}$.

Thus, equal heating rates correspond to identical stresses in both the cases examined and in all intermediate cases, and from the point of view of allowable thermal stresses in the plate, the heating rate does not depend on whether it is heated from one or from both sides.

The examined model of thermal stresses in an infinite plate gives a more or less accurate indication of the longitudinal (axial) stresses in the flange, but it is completely unsuitable for evaluating the stresses on the surfaces parallel to the joint, and especially at the joint itself, where there can be no tensile stresses.

The compressive stresses in the joint are counterbalanced for each half of the flange by the force exerted by the clamp (pin or bolt), and, since the clamp has a prestress, compressive stresses are set up over the whole surface of contact of the two halves of the flange if the temperature distribution is uniform.

Because of the great stiffness of the flange in bending, we may clearly neglect its deflection, and assume, as is usual in flange calculations, that the stresses in the joint vary linearly. When there is a temperature nonuniformity in the flange, we have a combination of the stresses exerted by the clamp and thermal stresses.

For symmetrical heating

$$\sigma = \frac{\beta E}{1-\nu}\left[-\frac{kx^2}{2} + \frac{kR^2}{6} + cx + \sigma_0\right], \tag{2}$$

where σ_0 and c are independent of the coordinates.

Equation (2) retains its form, by and large, for the stressed part of the surface of the joint, even when the stresses at some section fall to zero in connection with the increase in temperature nonuniformity.

To form the equilibrium equation of the flange at an arbitrary time, it is necessary to integrate the stresses and the corresponding moments, the limits of integration varying in accordance with the variation of the boundary of the stressed zone of the joint.

In practice, the attempt to solve the problem in this way encounters unforeseen mathematical difficulties. The solution may be obtained simply enough if the region of integration is constant. This condition may be considered satisfied for flanges with a recess in the joint. Beyond the limits of the recess there can be no stresses only when the nonuniformity of the temperature field is extremely great.

The assumption that the region of integration coincides with the unrecessed part of the joint may then lead only to some overestimate of the calculated values of the stresses in the flange and to some underestimate of them in the bolt, compared to the actual values.

The conditions of static equilibrium for a section of flange equal in length to the step may be written as

$$\int_{-R}^{x_-} \sigma\, dx + \int_{x_+}^{R} \sigma\, dx = -\frac{S}{h}\sigma_{\text{b}}, \tag{3}$$

$$\int_{-R}^{x_-} \sigma\, x dx + \int_{x_+}^{R} \sigma\, x dx = -\frac{S}{h}\sigma_{\text{b}}\, x_{\text{b}}. \tag{4}$$

Here no account has been taken of the moment acting on the flange from the wall side, this being customary in calculations of flanged connections [3]. The load on the connection due to excess pressure in the cylinder has also been neglected in formulating (3) and (4). This pressure, which increases somewhat the stresses in the bolt and at the outside edge of the joint and relieves the most highly stressed inside edge, does not play a significant part during heating.

To determine the temperature field on the flange, required for evaluation of the stresses in it, we equate the heat flux through unit surface of the flange in unit time from both sides

$$dq = \lambda \left.\frac{\partial t}{\partial x}\right|_{x=R} - \lambda \left.\frac{\partial t}{\partial x}\right|_{x=-R}$$

and the rate of increase of heat content of the corresponding part of the flange

$$dq = c_{\text{f}}\gamma \frac{\partial}{\partial \tau}\left(\int_{-R}^{R} t dx\right). \tag{5}$$

Expression (1), which gives the temperature field for a plate and does not take into account the bolt holes in the flanges, which impede the propagation of heat, may, however, be used without risk of great error under conditions of symmetrical heating when the heat flux in the drilled region is small. In this case, when c_{Φ}, λ, and γ are constant

$$kR\lambda = c_{\text{f}}\gamma \frac{\partial}{\partial \tau}\left(\int_{0}^{R} t\, dx\right).$$

The heat flux from the walls, which has practically no effect on the temperature of the inside edge of the joint, may increase the mean integral temperature of the flange and bolt and lower the stresses somewhat.

Similarly, we may determine the temperature conditions of the bolt for the case when the only heat flowing into it comes from the flange. Neglecting the very small temperature nonuniformity in the bolt, we have

$$\alpha\,(t_f - t_b) = \frac{d\,\gamma_b\,c_b}{4}\,\frac{dt_b}{d\tau}. \tag{6}$$

The stresses in the bolt are determined by its prestress σ_s and the temperature difference between it and the flange; the characteristic temperature to be used for high flanges, for which the hypothesis of plane sections holds, is the mean integral temperature

$$\bar{t} = \frac{1}{2R}\int_{-R}^{R} t\,dx.$$

Under the above assumptions

$$\bar{t} = t_0 + f(\tau) + \frac{kR^2}{6},$$
$$\sigma_b = \sigma_3 + k_c\,\beta_b\,E_b\,(\bar{t} - t_b). \tag{7}$$

The system of equations (7) contains eight unknown functions: t, σ, t_b, c, f, σ_b, k, σ_0, the first two of which depend on the coordinate and time, and the remainder only on time. To obtain a unique solution for a known initial condition of the system, we must impose an additional restraint.

The rate of heating may be limited by stretching of the bolt, buckling of the joint, or longitudinal stresses on the inside of the flange. It is therefore convenient to assume as the additional condition that the most dangerous of these quantities is maintained at its maximum allowable value, which in the simplest case is the constant level σ_a. With this condition we can find the remaining stresses, and, if necessary, replace the additional condition. One can also visualize the case when the initial heating is determined by one additional condition, and then another parameter becomes the determining one. Then the conditions at the end of the first heating period will be the initial values for the second.

In calculations carried out for two turbines, the limitation proved to be the stresses in the joint. Then the additional condition has the form

$$\sigma_a = \sigma\,|_{x=R} = \frac{\beta E}{1-\nu}\left(-\frac{kR^2}{3} + \sigma_0 + cR\right). \tag{8}$$

The system of equations (1)-(8) can be solved in closed form for parametric linearization of the equations (all physical properties of the materials and the heat transfer coefficient from flange to bolt are assumed constant) and with the other assumptions made above.

Introducing the notation

$$4\alpha/c_b\,\gamma_b\,d = q;\ \ \lambda/c\,\gamma = a;\ \ \sigma_a(1-\nu)/\beta E = \sigma_1,$$

we can write equation (6) as

$$t_b' + qt_b = qt_f.$$

The solution of this equation has the form

$$t_b = \exp(-q\tau)\,\left[\int_0^{\tau} qt_f \exp(q\tau)\,d\tau + C_1\right].$$

In the case of symmetrical heating of the flange, the expression for t_p, following substitution of t from (1), at the level of the bolt may be written thus:

$$t_b = q\exp(-q\tau)\int_0^{\tau}\exp(q\tau)\,f(\tau)\,d\tau + t_0 + C_1\exp(-q\tau) +$$
$$+\exp(-q\tau)\,\frac{x_b^2}{2}\,q\int_0^{\tau} k\exp(q\tau)\,d\tau. \tag{9}$$

From (1) and (5) we find

$$f(\tau)=a\int_0^{\tau}kd\tau-t_0-\frac{kR^2}{6}$$

and, substituting $f(\tau)$ into (9), we finally obtain

$$t_{\text{b}}-\bar{t}=C_1\exp(-q\tau)-\left(a+q\frac{R^2}{6}-qx_{\text{b}}^2\frac{1}{2}\right)\times \times\exp(-q\tau)\int_0^{\tau}k\exp(q\tau)\,d\tau, \tag{10}$$

where $K=\int_0^{\tau}k\,d\tau,\ K'=k.$

Substituting this value of $t_p - t$ into (7), and equating the σ_p from (3) and (7), we obtain the integral equation

$$a_1+a_2k+a_3c=a_5\exp(-q\tau)\int_0^{\tau}k\exp(q\tau)\,d\tau+a_4\exp(-q\tau), \tag{11}$$

where k and c are unknown functions of time

$$a_1=2\sigma_1R-\sigma_1(x_+-x_-)+\frac{S}{h}\sigma_3\frac{1-\nu}{\beta E},$$

$$a_2=\frac{2}{3}R^3-\frac{R^2}{2}(x_+-x_-)+\frac{1}{6}(x_+^3-x_-^3),$$

$$a_3=-2R^2+R(x_+-x_-)-\frac{1}{2}(x_+^2-x_-^2),$$

$$a_4=\frac{S}{h}(1-\nu)C_1,$$

$$a_5=-(1-\nu)\frac{S}{h}\left(a+q\frac{R^2}{6}-q\frac{x_{\text{b}}^2}{2}\right)k_n.$$

Equating the left sides of (3) and (4), having first eliminated σ, we use (2) to obtain a second linear equation

$$b_1k+b_2=b_3c, \tag{12}$$

where

$$b_1=\frac{2}{3}R^3x_{\text{b}}-\frac{R^2x_{\text{b}}}{2}(x_+-x_-)+\frac{x_{\text{b}}}{6}(x_+^3-x_-^3)+ +\frac{1}{8}(x_+^4-x_-^4)+\frac{R^2}{4}(x_+^2-x_-^2),$$

$$b_2=2\sigma_1Rx_{\text{b}}-\sigma_1x_{\text{b}}(x_+-x_-)+\frac{\sigma_1}{2}(x_+^2-x_-^2),$$

$$b_3=\frac{2}{3}R^3+\frac{R}{2}(x_+^2-x_-^2)-\frac{1}{3}(x_+^3-x_-^3)+2R^2x_{\text{b}}- -Rx_{\text{b}}(x_+-x_-)+\frac{x_{\text{b}}}{2}(x_+^2-x_-^2).$$

Solving (11) and (12) simultaneously, we obtain an integral equation for k, and, putting

$$\frac{a_1+a_3b_2/b_3}{a_2+a_3b_1/b_3}=B,\quad\frac{a_5}{a_2+a_3b_1/b_3}=H,\quad\frac{a_4}{a_2+a_3b_1/b_3}=A,$$

$$\int_0^\tau k \exp(q\tau)\, d\tau = \varkappa, \ \ \varkappa' = k \exp(q\tau),$$

we obtain the differential equation

$$\varkappa' - H\varkappa = A - B\exp(q\tau),$$

the solution of which is

$$\varkappa = -\frac{A}{H} + \frac{B}{H-q} \exp(q\tau) + C_2 \exp(H\tau).$$

Hence

$$k = C_3 \exp[(H-q)\tau] - \frac{Bq}{q-H}, \tag{13}$$

$$c = \frac{b_2}{b_3} + \frac{b_1}{b_3} C_3 \exp[(H-q)\tau] - \frac{Bq}{q-H}\frac{b_1}{b_3}. \tag{14}$$

When $\tau = 0$

$$t_b = t_0, \ \ t_b - \bar{t} = -kR^2/6,$$

$$C_1 = \left(a + q\frac{R^2}{6} - q\frac{x_b^2}{2}\right)\left(\frac{C_3}{H} - \frac{B}{q-H}\right) - \frac{kR^2}{6}, \tag{10a}$$

$$a_1 + a_2\left(C_3 - \frac{Bq}{q-H}\right) + a_3\left(\frac{b_2}{b_3} + \frac{b_1}{b_3}C_3 - \frac{Bq}{q-H}\frac{b_1}{b_3}\right) = $$

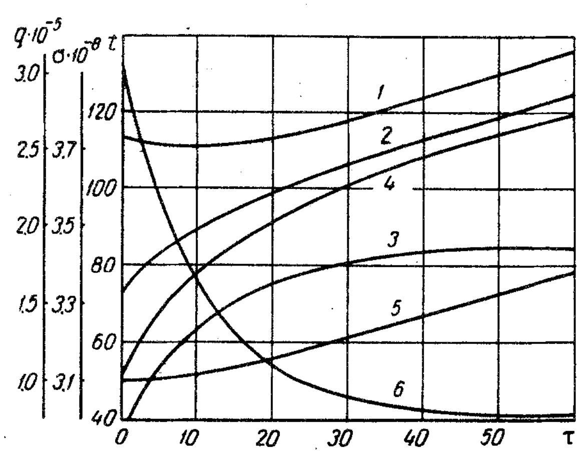
$$= a_5\left(\frac{C_3}{H} - \frac{B}{q-H}\right) + \frac{S}{h}(1-\nu)C_1. \tag{11a}$$

Equations (10a) and (11a) determine C_1 and C_3. After substitution of C_1 into (11a), we obtain

$$a_1 + a_2^0\left(C_3 - \frac{Bq}{q-H}\right) + $$
$$+ a_3\left(\frac{b_2}{b_3} + \frac{b_1}{b_3}C_3 - \right.$$
$$\left. - \frac{Bq}{q-H}\frac{b_1}{b_3}\right) = 0,$$

where

$$a_2^0 = a_2 + \frac{S}{h}(1-\nu)\frac{R^2}{6}.$$



Some results of heating calculations for the flanged connection of a cylindrical turbine: 1) $t_{x=R}$; 2) t; 3) σ_h; 4) $t_{x=0}$; 5) t_h; 6) q.

Some results of heating calculations for the flanged connection of one of the turbines are given in the figure.

The calculated longitudinal compressive stresses on the inside face of the flange during the whole heating period prove to be lower than the stresses in the joint. The actual stresses will be even lower due to weakening of the flange by the bolt holes. It should be noted that the above calculation gives the maximum allowable heating rate of the flange at each moment of time, which may be unattainable in practice.

However, one can check that the stresses produced are allowable by adding the additional condition corresponding to the actual heating regime to the system of equations (1)-(7).

NOTATION

Characteristics of the flange: E – modulus of elasticity of material; β – coefficient of linear expansion of material, c_f – specific heat; γ – specific weight of material; λ – thermal conductivity; $f(\tau)$ and R – functions of time; x_{min} – coordinate of minimum of temperature curve (for symmetrical heating it vanishes), equal to (–R) if the plate receives heat on the side where x is positive and is insulated on the opposite side (unilateral heating); k_c – a coefficient, taking into account compliance; ν – Poisson's ratio; R – characteristic dimension; x_+, x_- – coordinates of edges of recess; $t_{x=R}$ and $t_{x=0}$ – temperatures at inside and in middle of flange; t_0 – initial temperature of flange; σ – stress in flange; α – reduced coefficient of heat transfer from flange to bolt, taking account of heat flux through contact between them; q – heat flux to flange. Characteristics of the bolt: E_p – modulus of elasticity of material; β_b – coefficient of linear expansion of material; c_b – specific heat; γ_b – specific weight of material; h – pitch of bolt; S – cross-sectional area; σ_b – stress in bolt; t_b – temperature of material; d – diameter of bolt; x_b – coordinate of bolt axis.

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